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Double-diffusive convection with an imposed magnetic field

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Abstract—The linear and finite amplitude two-dimensional double-diffusive magnetoconvection (thermohaline convection in the presence of a magnetic field) has been studied analytically with free horizontal boundaries held at fixed temperature and concentration. It is shown that the magnetic field acts as a third diffusing component and its effect is to suppress convection. In the case of linear theory the conditions for direct and oscillatory modes are obtained and the stability boundaries for salt-finger and double-diffusive convection are predicted in the Rayleigh number plane. If τ_2, the ratio of magnetic diffusivity to thermal diffusivity, is small and the solute Rayleigh number R_s and Chandrasekhar number Q are sufficiently large convection sets in as overstable oscillations and the onset of it is approximated by two straight lines in the Rayleigh number plane. It is found that the salt-finger and overstable modes may be simultaneously unstable over a wide range of conditions and the effect of the magnetic field is to suppress this region. In the case of nonlinear theory it is found that the finite amplitude magnetoconvection exists for subcritical values of the Rayleigh number R, for all Q and τ_1 (which is the ratio of solute diffusivity to the thermal diffusivity) when $\tau_2 = 0.1$ and $R_s = 10^4$. It is found that the heat transport increases with an increase in R and decrease in τ_2 but decreases with Q.

1. INTRODUCTION

DOUBLE-diffusive convection in an ordinary viscous flow has been extensively investigated, both theoretically and experimentally, for about a quarter of a century [1–11] because of its various applications [12]. Magnetoconvection (convection in the presence of a magnetic field) in a single component conducting fluid considering two additional effects beyond those present in non-MHD situations, namely that each unit volume of fluid experiences a body force $\mu \mathbf{J} \times \mathbf{H}$ and energy loss (due to Ohmic dissipation) of $\mathbf{J} \cdot \mathbf{J}/\sigma'$ where $\mathbf{H}$ is the magnetic field, $\mathbf{J}$ the current density, μ the magnetic permeability and σ' the electrical conductivity, has also been investigated by many authors [13–21] because of its importance in many engineering, geophysical and astrophysical applications. But, not much attention has been given to the interaction between the magnetic field and double-diffusive convection, called double-diffusive magnetoconvection, where the magnetic field acts as a third diffusing component. The study of this is the object of the present paper because of its importance in many fields of engineering, for example MHD generators [22] and astrophysics particularly in explaining the properties of large stars with a helium rich core [12, 23, 24].

We shall see that in a double-diffusive magnetoconvection the quiescent state breaks down and convective motion arise when the buoyancy force is sufficient to overcome the dissipative effects. The combined effect of magnetic field and salinity gradient is to suppress, both in infinitesimal and finite amplitude, steady convective motions depending on the strength of the magnetic field, the ratios of the diffusivity of salt to heat (i.e. τ_1) and magnetic diffusivity to heat diffusivity (i.e. τ_2). For other values of these parameters, oscillatory (i.e. overstable) and finite amplitude motions are possible.

The finite amplitude, steady double-diffusive magnetoconvection in a horizontal layer permeated by a transverse magnetic field was originally discussed by Lortz [25] with the motive of clarifying some of the mathematical aspects of the so-called 'relative stability' criterion of Malkus and Veronis [26]. Lortz [25] has discussed only some general properties of the solutions without actually solving the differential equations. Although, the Lortz [25] analysis provides the behaviour of various steady solutions, it is silent about the prediction of different kinds of stability and heat transport and the effect of magnetic field on them. The prediction of these aspects is the main object of this paper. In the case of linear theory we shall see that the effect of magnetic field upon double-diffusive magnetoconvection is to act as a third diffusing component as in non-MHD flow discussed by Griffiths [3]. In the case of linear theory both magnetic field and solute concentration make a linear contribution to the equation of motion. However, the situation is quite different and greater variety of behaviour is possible in the nonlinear theory because of the quadratic restoring Lorentz force in the momentum equation.

The plan of work of this paper is as follows. The basic equations and the corresponding boundary conditions are discussed in the next section. This is followed by the discussion of linear theory. The conditions for the onset of direct (i.e. marginal) and oscillatory (i.e. overstable) convection are determined in Section 3 and the stability boundaries for salt-finger (warm salty water overlies

NOMENCLATURE

d	vertical length scale
$\mathbf{g}$	acceleration due to gravity
$\mathbf{H}$	magnetic field, (H_x, H_y, H_z)
H	total heat transport
H_0	imposed magnetic field
$J(f, g)$	Jacobian, $\partial(f, g)/\partial(x, z)$
$\mathbf{J}$	current density
$\mathbf{k}^2$	$\pi^2(\alpha^2+1)$
Nu	Nusselt number
Nu_s	solute Nusselt number
p	frequency, $p_r + i\, p_i$
Q	Chandrasekhar number
R	thermal Rayleigh number
R_s	solute Rayleigh number
S	solute concentration
ΔS	concentration difference between lower and upper layers
T	temperature
ΔT	temperature difference between lower and upper layers
t	time coordinate
u, w	horizontal and vertical velocity components
x, y, z	horizontal and vertical space coordinates
∇^2	$\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}$.

Greek symbols

α	horizontal wave number
α_T	thermal expansion coefficient
β	solute analog of α_T
κ	thermal diffusivity
κ_s	solute analog of κ
$(\lambda, \lambda_s, \lambda_m)$	$\left(\dfrac{\pi^2\alpha^2}{k^6}R, \dfrac{\pi^2\alpha^2}{k^6}R_s, \dfrac{\tau_2 Q\pi^2}{k^4}\right)$
μ	magnetic permeability
ν	kinematic viscosity
ν_m	magnetic viscosity
ρ, ρ_m	density and mean density of the fluid
σ	Prandtl number, ν/κ
σ_m	magnetic Prandtl number, ν/ν_m
τ_1	ratio of diffusivities, κ_s/κ
τ_2	ratio of magnetic to thermal diffusivity, ν_m/κ
ϕ	magnetic stream function
ψ	stream function
ω	frequency, $p/k^2 = \omega_r + i\,\omega_i$.

cooler fresh water) and double-diffusion convections (lower layer is warmer and more saline than upper layer) are predicted in Section 4 for different values of the magnetic parameters. The effect of varying α for fixed values of magnetic parameters on stability boundaries is discussed in Section 5. This linear theory predicts only the conditions for the onset of convection and is silent about the prediction of heat transport which is the realm of nonlinear theory. Therefore, to know the effects of the magnetic field and the salinity gradient on the heat transport the finite amplitude double-diffusive magnetoconvection is discussed in Section 6 using a truncated representation of Fourier series for dependent variables as in Veronis [27] and Rudraiah *et al.* [28]. Though this is very restrictive, it is useful to understand the physics of this complicated problem with minimum mathematics. The results of this study will also serve as the starting values for the discussion of the general non-linear system using numerical experiments. The final section summarizes the important results.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider a horizontal layer of infinite extent filled with a Boussinesq electrically conducting fluid, subject to an adverse temperature gradient ΔT, stabilizing salinity gradient ΔS and permeated by an externally applied uniform magnetic field H_0 normal to the layer. The Cartesian coordinate system has been taken with the origin in the lower boundary, and the z-axis vertically upwards. The boundaries are assumed to be dynamically free in the absence of surface tension and are also perfect conductors of heat and solute. The upper plate $z = d$ is at a constant temperature $(T_m - \Delta T)$ and constant concentration $(S_m - \Delta S)$; while the lower plate $z = 0$ is at temperature T_m and concentration S_m. It is convenient, in the following analysis, to break up the temperature and solute into two parts; one the linear part given above and the other part due to convective redistribution. Thus

$$
\begin{aligned}
T_{\text{total}} &= T_m - \Delta T\, z/d + T(x, y, z, t) \\
S_{\text{total}} &= S_m - \Delta S\, z/d + S(x, y, z, t)
\end{aligned}
\tag{1}
$$

with solute gradient stabilizing and temperature gradient destabilizing. We assume that all the physical quantities are independent of y, i.e. we consider only the two-dimensional horizontal rolls. The density is taken to be $\rho = \rho_m\{1-\alpha_T(T-T_m)+\beta(S-S_m)\}$ where T is the temperature, S is the solute concentration and α_T, $\beta > 0$.

By introducing the stream function ψ and the magnetic stream function ϕ, through

$$u = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial x}$$
$$H_x = \frac{\partial \phi}{\partial z}, \quad H_z = -\frac{\partial \phi}{\partial x} \tag{2}$$

and scaling the velocity with the thermal diffusivity κ, as κ/d, length by d, time by d^2/κ, magnetic field by imposed field H_0, temperature by ΔT and the concentration by ΔS we arrive at the following dimensionless equations:

$$\left(\frac{1}{\sigma}\frac{\partial}{\partial t} - \nabla^2\right)\eta = \tau_2 Q \frac{\partial \xi}{\partial z} - R\frac{\partial T}{\partial x} + R_s \frac{\partial S}{\partial x} + \frac{1}{\sigma} J(\psi, \eta) - \tau_2 Q J(\phi, \xi) \tag{3}$$

$$\left(\frac{\partial}{\partial t} - \tau_2 \nabla^2\right)\phi = \frac{\partial \psi}{\partial z} + J(\psi, \phi) \tag{4}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T = -\frac{\partial \psi}{\partial x} + J(\psi, T) \tag{5}$$

$$\left(\frac{\partial}{\partial t} - \tau_1 \nabla^2\right)S = -\frac{\partial \psi}{\partial x} + J(\psi, S) \tag{6}$$

where $\eta = \nabla^2\psi$ is the vorticity, $\xi = \nabla^2\phi$ is the current density and J stands for the Jacobian. The following non-dimensional parameters appear:

ratio of diffusivities:	$\tau_1 = \kappa_s/k$
Prandtl number:	$\sigma = \nu/\kappa$
magnetic Prandtl number:	$\sigma_m = \nu/\nu_m$
Chandrasekhar number:	$Q = \mu H_0^2 d^2/\rho_m \nu \nu_m$
ratio of the magnetic to thermal diffusivity:	$\tau_2 = \nu_m/\kappa$
Rayleigh number:	$R = \alpha_T g \Delta T d^3/\kappa\nu$
solute Rayleigh number:	$R_s = \beta g \Delta S d^3/\kappa\nu$.

Here, R_s is defined with κ rather than mass diffusivity κ_s in the denominator.

The boundary conditions for the problem are straight forward. Since the boundaries are assumed to be flat, magnetic and velocity stress free, the required boundary conditions are

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = T = S = \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = 0, 1. \tag{7}$$

Physically the conditions on magnetic field imply that the total flux in each cell is fixed, but the field lines are free to move horizontally along the upper and lower boundaries, though they must remain vertical there.

3. LINEAR STABILITY ANALYSIS

The basic equations for the linear stability problem are obtained when the Jacobian terms in equations (3)–(6) are neglected. In this case we look for the solution of these linear equations in the form

$$\psi \sim e^{pt} \sin \pi\alpha x \sin \pi z, \quad \phi \sim e^{pt} \sin \pi\alpha x \cos \pi z$$
$$T, S \sim e^{pt} \cos \pi\alpha x \sin \pi z \tag{8}$$

where p is the growth rate, $\pi\alpha$ is the horizontal wave number and π is the vertical wave number. Substituting these into the above linearized equations and after some simplification yields the characteristic equation

$$\begin{aligned}&\frac{\omega^4}{\sigma} + \frac{\omega^3}{\sigma}\{\sigma + 1 + \tau_1 + \tau_2\} \\ &+ \omega^2\{1 + \tau_1 + \tau_2 + \tau_1/\sigma + \tau_2/\sigma \\ &+ \tau_1\tau_2/\sigma - \lambda + \lambda_s + \lambda_m\} \\ &+ \omega\{\tau_1 + \tau_2 + \tau_1\tau_2 + \tau_1\tau_2/\sigma - (\tau_1 + \tau_2)\lambda \\ &+ (1 + \tau_2)\lambda_s + (1 + \tau_1)\lambda_m\} \\ &+ \tau_1\tau_2\{1 - \lambda + \lambda_s/\tau_1 + \lambda_m/\tau_2\} = 0\end{aligned} \tag{9}$$

with

$$\omega = p/k^2, \quad \lambda = \frac{\pi^2\alpha^2}{k^6} R, \quad \lambda_s = \frac{\pi^2\alpha^2}{k^6} R_s,$$
$$\lambda_m = \tau_2 Q \pi^2/k^4 \quad \text{and} \quad k^2 = \pi^2(\alpha^2 + 1).$$

Now $Q\tau_2$, like Rayleigh number, has the product $\kappa\nu$ in the denominator and is independent of ν_m. It is of interest to note that the final stability equation (obtained by eliminating all the dependent variables except one among equations (3)–(6)) will contain four time derivatives and hence the characteristic equation (9) is of the order of four in contrast to the three time derivatives contained in the stability equation for a double-diffusive convection in a viscous flow [29] or in porous media [28]. Although the dispersion relation (9) is analogous to the one given by Griffiths [3] for three component non-MHD system, the behaviour of this is entirely different. For example, the third diffusing component in a non-MHD system may act as stabilizing or destabilizing agent depending on the sign of the concentration gradient of the third component; whereas in the present problem the third diffusing component, namely the magnetic field, acts as a stabilizing agent. We note that equation (9) reduces to the dispersion relation given by Baines and Gill [29] when $\tau_2 = 0$.

3.1. *Marginal state*

There is a marginal state ($p_r = 0$ and $p_i = 0$) when $R = R^c$, where

$$R^c = R^m + R_s/\tau_1 \tag{10}$$

and

$$R^m = \frac{\pi^2(\alpha^2+1)}{\alpha^2}\{\pi^2(\alpha^2+1)^2 + Q\} \tag{11}$$

is the value of R^c in the single component magnetoconvection (i.e. $R_s \to 0$ or $\tau_1 \to \infty$) given by Chandrasekhar [13], provided $\alpha^2 = x$ satisfies the equation

$$2x^3 + 3x^2 = 1 + Q/\pi^2. \tag{12}$$

It is of interest to note that equation (12) for the critical wave number is independent of R_s and τ_1 and hence is identical with the one given by Chandrasekhar [13] for a single component conducting fluid. Although the magnetic field and vertical concentration gradient stabilizes the system by balancing with the usual buoyancy force, the cell pattern will be affected only by the constraint of magnetic field and not by the concentration gradient. The plane surface of a marginal stability on which relation (10) holds will be denoted by $\mathscr{P}$.

3.2. *Overstable state*

It is well known that the overstable motion is possible only in the presence of the external constraints like rotation [27, 28] salinity gradient [1, 8–10, 30] and magnetic field [13, 17, 20, 31] and the physical explanation for the existence of overstable (i.e. oscillatory) motion is also well known. The same physical argument is true for causing oscillatory motion in a double-diffusive magnetoconvection where the salinity gradient and magnetic field act as additive effects. This is discussed in detail in this section.

For the oscillatory mode $p_r = 0$, $p_i \neq 0$, i.e. $p = \mathrm{i}\, p_i$ where p_i has to be real. Substituting $\omega = \mathrm{i}\,\omega_i$ in equation (9) and equating the real and imaginary parts to zero we have

$$\frac{\omega_i^4}{\sigma} - \omega_i^2(D-\lambda+\lambda_s+\lambda_m) + \tau_1\tau_2\left(1-\lambda+\frac{\lambda_s}{\tau_1}+\frac{\lambda_m}{\tau_2}\right) = 0 \quad (13)$$

$$\omega_i^2 = \frac{\sigma}{B}\{A-(\tau_1+\tau_2)\lambda+(1+\tau_2)\lambda_s+(1+\tau_1)\lambda_m\} \quad (14)$$

provided $\omega_i \neq 0$, where

$$A = \tau_1+\tau_2+\tau_1\tau_2(1+\sigma^{-1})$$
$$B = \sigma+1+\tau_1+\tau_2$$
$$D = 1+\tau_1+\tau_2+\sigma^{-1}(\tau_1+\tau_2+\tau_1\tau_2).$$

Together, equations (13) and (14) describe the curve

$$a_1\lambda^2+a_2\lambda_s^2+a_3\lambda_m^2+a_4\lambda\lambda_s+a_5\lambda\lambda_m + a_6\lambda_s\lambda_m+a_7\lambda+a_8\lambda_s+a_9\lambda_m+a_0 = 0 \quad (15)$$

where the coefficients are

$$a_1 = -\{\tau_1+\tau_2\}\{1+\sigma\}$$
$$a_2 = -\{\sigma+\tau_1\}\{1+\tau_2\}$$
$$a_3 = -\{\sigma+\tau_2\}\{1+\tau_1\}$$
$$a_4 = -2\{\tau_1+\tau_2\}\{1+\tau_2\}+B\{1+\tau_1+2\tau_2\}$$
$$a_5 = -2\{\tau_1+\tau_2\}\{1+\tau_1\}+B\{1+2\tau_1+\tau_2\}$$
$$a_6 = 2\{1+\tau_1\}\{1+\tau_2\}-B\{2+\tau_1+\tau_2\}$$
$$a_7 = \{BD-2A\}\{\tau_1+\tau_2\}+AB-\tau_1\tau_2B^2\sigma^{-1}$$
$$a_8 = -\{BD-2A\}\{1+\tau_2\}-AB+\tau_2B^2\sigma^{-1}$$
$$a_9 = -\{BD-2A\}\{1+\tau_1\}-AB+\tau_1B^2\sigma^{-1}$$
$$a_0 = A^2-ABD+\tau_1\tau_2B^2\sigma^{-1}.$$

For oscillatory motions we require $\omega_i^2 > 0$ in equations (13) and (14). This is true only if

$$(\tau_1+\tau_2)\lambda-(1+\tau_2)\lambda_s-(1+\tau_1)\lambda_m \leqslant A \quad (16)$$

and

$$D-\lambda+\lambda_s+\lambda_m \pm G^{1/2} \geqslant 0 \quad (17)$$

where

$$G = \lambda^2+\lambda_s^2+\lambda_m^2+2(-\lambda\lambda_s+\lambda_s\lambda_m-\lambda\lambda_m) + 4\tau_1\tau_2\sigma^{-1}(\lambda-\lambda_s\tau_1^{-1}-\lambda_m\tau_2^{-1}) - 2D(\lambda-\lambda_m-\lambda_s)+D^2-4\tau_1\tau_2\sigma^{-1}.$$

Equation (15) together with conditions (16) and (17) represent different surfaces depending on the values of τ_1 and τ_2. For example, for $\tau_1 > 0$ and $\tau_2 < 1$, equation (15) represents an hyperboloid denoted by $\mathscr{H}$, of which inequalities (16) and (17) select the appropriate branch giving rise to the conditions at which the most unstable oscillatory mode is marginally stable. On the other hand, in the case of a single component magnetoconvection i.e. $\tau_1 = \tau_2 = 1$, (15) represents a paraboloid. Further the intersection of (15) with any plane parallel to the plane $\lambda = 0$ becomes parabolic when $\tau_1, \tau_2 = 0$.

The plane boundary $\mathscr{P}$ and the surface $\mathscr{H}$ alone still do not determine fully the conditions under which marginal or oscillatory convection may occur because there may be a possibility that the complex roots of the quartic characteristic equation (9) may become real roots without their parts passing through zero [3]. Therefore, in the next section the possible positions of the boundaries $\mathscr{P}$ and $\mathscr{H}$ are illustrated for suitable values of the parameters τ_1, τ_2 and Q.

4. STABILITY BOUNDARIES FOR DIFFERENT MAGNETIC PARAMETERS

In this section the stability boundaries are discussed by taking specific examples based on the values of τ_1, τ_2 and Q.

4.1. *A case where τ_1 and $\tau_2 \leqslant 1$*

We consider the values of τ_1 and τ_2 covering the range of interest both in geophysical and in laboratory models where the diffusivity ratios are not far from unity. For example in molten metals and magma in geophysical problems and KCl, NaCl, liquid sodium solutions in the laboratory have diffusivity ratios not far from unity. We define the plane $\mathscr{S}$ given by

$$\lambda = \lambda_s+\lambda_m. \quad (18)$$

By choosing $\tau_1 = 0.81, 0.32$, $\tau_2 = 0.75, 0.5, 0.25$ and $Q = 10, 10^2$ the relevant portions of the intersections of $\mathscr{P}$, $\mathscr{H}$ and $\mathscr{S}$ are drawn in Figs. 1(a) and (b). The 'relevant portions' mean those which describe a change in the mode of instability for the most unstable mode. In these figures the horizontally hatched regions give oscillatory modes and the oblique hatching shows

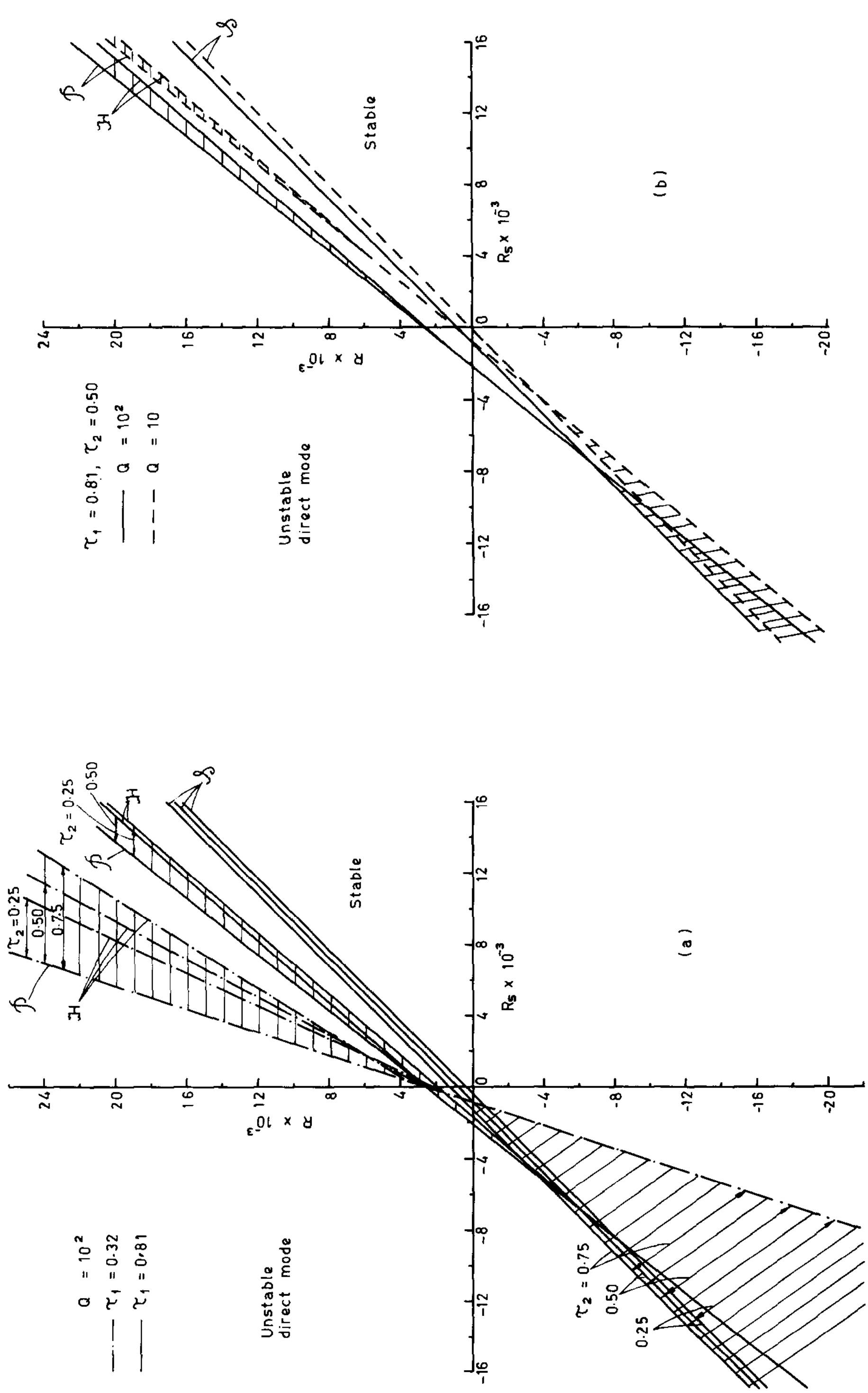

FIG. 1. Stability boundaries for different values of Q, τ_2 and τ_1. Horizontally hatched regions give overstable modes. Oblique hatched regions show conditions unstable to salt-fingers.

conditions unstable to salt-fingers. From Fig. 1(a) it is clear that smaller τ_1 diffuses more salt and also the region of overstable modes increases as τ_2 decreases provided $\tau_1 > \tau_2$ and this region increases as τ_2 increases when $\tau_1 < \tau_2$.

To know the effect of the magnetic field, we have drawn in Fig. 1(b) the stability boundaries for different Q. It is clear that the region of stability increases as Q increases and thus establishing that the effect of magnetic field is to inhibit the onset of convection. The asymptotes of the hyperbola are almost parallel, the slope of each being independent of the values of Q and τ_2. Salt-fingers and overstable modes can occur (as the most unstable mode) at adjacent condition and more extensively when the combined effects of temperature and magnetic field components are stabilizing with $\tau_2 = 0.25$ (third quadrant of Fig. 1(a)).

4.2. *A case where* $\tau_1, \tau_2 \ll 1$

The discussion of a double-diffusive magnetoconvection may be simplified when $\tau_1, \tau_2 \to 0$ and $\sigma \gg \tau_1$, since $\mathscr{H}$ approaches its asymptotes in this limit and the asymptotes themselves degenerate into a pair of planes given by

$$R = \frac{\sigma}{\sigma+1}\left(R_s + \lambda_m \pi^4 \frac{(\alpha^2+1)^3}{\alpha^2}\right) + \pi^4 \frac{(\alpha^2+1)^3}{\alpha^2} \tag{19}$$

$$R = (\tau_1+\tau_2)^{-1}\left(R_s + \lambda_m \pi^4 \frac{(\alpha^2+1)^3}{\alpha^2}\right) + \pi^4 \frac{(\alpha^2+1)^3}{\alpha^2} \tag{20}$$

and they satisfy the equations

$$2x^3 + 3x^2 = 1 + \frac{\sigma}{\sigma+1}\frac{Q\tau_2}{\pi^2} \tag{21}$$

$$2x^3 + 3x^2 = 1 + (\tau_1+\tau_2)^{-1}\frac{Q\tau_2}{\pi^2} \tag{22}$$

with $x = \alpha^2$. Although the double-diffusive magnetoconvection is analogous, in linear analysis, to the non-MHD three component convection [3] there are some important differences too. One such difference is the value of the critical wave number (i.e. cell pattern). In the non-MHD case Griffiths [3] has used the critical wave number $\alpha_c^2 = \frac{1}{2}$. In the double diffusive magnetoconvection, however, the critical wave number given by equations (21) or (22) varies (i.e. change in cell pattern) with Q, τ_1, σ and τ_2. In other words we cannot fix the critical wave number in the double-diffusive magnetoconvection as in the non-MHD three component case discussed by Griffiths [3].

Figures 2(a) and (b) show the relevant portions of the intersections of $\mathscr{P}$, $\mathscr{H}$ and $\mathscr{S}$ for different values of Q and τ_2 ($Q = 10^3, 10^4$ and $\tau_2 = 0.025, 0.0025, 0.005$) and for fixed values of τ_1 ($=0.01$) and σ ($=7.0$). It can be seen that $\mathscr{H}$ is described very closely by equations (19) and (20). To depict the three-dimensional geometry more clearly the intersections of the surfaces $\mathscr{P}$ and $\mathscr{S}$ as functions of Q and τ_2 will also be defined and are shown as dotted lines in Fig. 2(b). The point of intersections of the asymptotes always lies to the right of $\mathscr{P}$ (see Fig. 2(b)). Then the lower asymptote, equation (20), of $\mathscr{H}$ and plane $\mathscr{P}$ converge slowly to intersect at P, a point where

$$R = Q\pi^2\left\{\frac{\alpha_1^2+1}{\alpha_1^2} - \frac{1}{\tau_3}\frac{(\alpha_2^2+1)}{\alpha_2^2}\right\} + \frac{\pi^4}{\tau_3}\left\{(1+\tau_3)\frac{(\alpha_1^2+1)^3}{\alpha_1^2} - \frac{(\alpha_2^2+1)^3}{\alpha_2^2}\right\} \tag{23}$$

$$R_s = Q\pi^2\left\{\tau_1\frac{(\alpha_1^2+1)}{\alpha_1^2} - \frac{(\tau_1+\tau_2)}{\tau_3}\frac{(\alpha_2^2+1)}{\alpha_2^2}\right\} + \pi^4\frac{(\tau_1+\tau_2)}{\tau_3}\left\{\frac{(\alpha_1^2+1)^3}{\alpha_1^2} - \frac{(\alpha_2^2+1)^3}{\alpha_2^2}\right\}. \tag{24}$$

Here α_1 and α_2 are the critical wave numbers of equations (19) and (20) and $\tau_3 = \tau_2/\tau_1 = \nu_m/\kappa_s$. The almost vertical lines in the third quadrant of Fig. 2(b) are the loci of P as Q and τ_2 are varied. Point P lies well within the region of static stability, so that the effect of magnetic field with a minimum τ_2 ($\tau_1 > \tau_2$) causes a large extension of the range of values of R and R_s at which overstable modes occur. But this is not the case when $\tau_1 < \tau_2$, even though the effect of $Q\tau_2$ is the same in both cases (see Fig. 2(a)).

5. EFFECT OF VARYING α FOR FIXED Q AND τ_2 ON THE DOUBLE-DIFFUSIVE MAGNETOCONVECTION

In the previous sections we have discussed the situation where α varies with Q and τ_2. However, to gain more physical insight it is advantageous to vary α by fixing the values of Q and τ_2. This gives another important difference between double-diffusive magnetoconvection and non-MHD triple diffusive convection [3]. In the non-MHD case [3] the plane surface $\mathscr{S}$, which was the locus of the intersections of $\mathscr{H}$ and $\mathscr{P}$ as α was allowed to vary, was vertical because R_s (in the notation of this paper) was independent of α. In this problem, however, equations (23) and (24) show that R_s depends on α and hence the plane surface $\mathscr{S}$ is not vertical as demonstrated below. The portion of the plane $\mathscr{P}$ above its intersection with $\mathscr{H}$ (at P in Fig. 2(b)) must now divide the conditions to the right at which only oscillatory disturbances are unstable from the conditions to the left at which both overstable and monotonically growing modes are possible. This behaviour is illustrated on the R_s–R plane in Fig. 3 for $Q = 10^4$, $\tau_2 = 0.0025$, $\sigma = 7$ and $\tau_1 = 0.01$.

In Fig. 3 another plane surface $\mathscr{L}$ has been defined, to the left of which no wavelengths are overstable only monotonic instability is possible. In the absence of magnetic field, Baines and Gill [29] found the appropriate line to be $R = [(\sigma+\tau_1)/(\sigma+1)\tau_1^2]R_s$. To explore the effect of magnetic field further we consider a system which lies in the third quadrant of Fig. 3 and

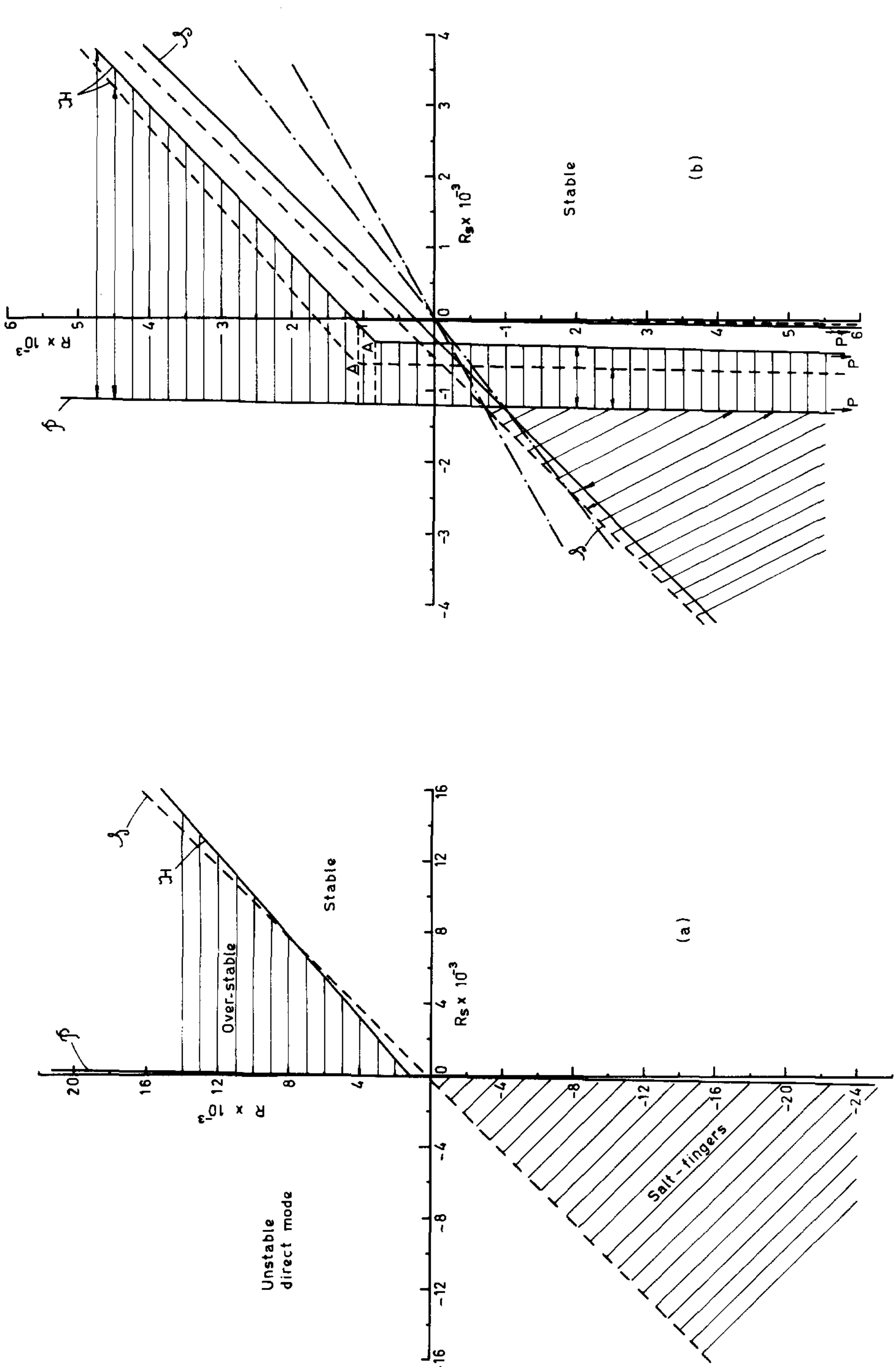

FIG. 2. Stability boundaries for the most unstable mode when $\tau_1 = 0.01$ and $\sigma = 7$ at two values of Q: (a) $Q = 10^3$, $\tau_2 = 0.025$; (b) $Q = 10^4$, $\tau_2 = 0.0025$ and $Q = 10^4$, $\tau_2 = 0.005$ (shown dashed). Hatching and heavy lines have the same meaning as in Fig. 1 and the lines are explained in the text.

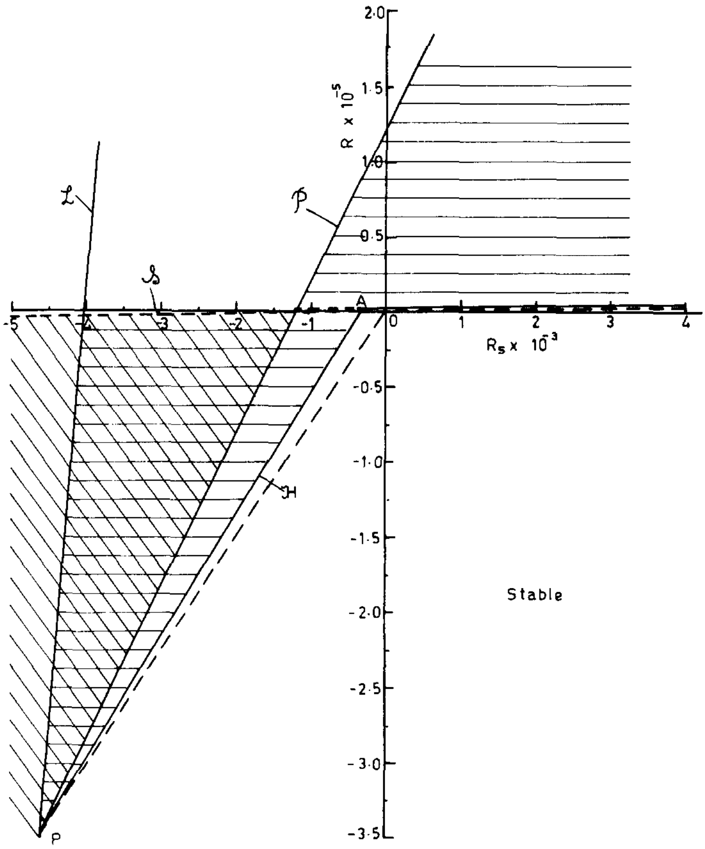

FIG. 3. Stability bounds of double-diffusive magnetoconvection for $Q = 10^4, \tau_2 = 0.0025, \sigma = 7$ and $\tau_1 = 0.01$. Both oscillatory and salt-finger modes are unstable in the double hatched region.

which always has $\partial\rho/\partial z < 0$. Then, the combined effect of temperature and magnetic field is opposite to that of salinity. When the negative R_s is sufficiently small the system is stable. However, if negative R_s is increased until the $(R, -R_s, Q\tau_2)$ coordinates cross $\mathscr{H}$ then some wavelengths become overstable beginning at $\alpha^2 = 4.218$, the mode which represents a balance between more efficient thermal diffusive, viscous and ohmic damping. Even though the oscillatory modes transport components with greater diffusivity more rapidly here they can transport more concentration by molecular diffusion than the combined heat and magnetic field. If R_s is more destabilizing and the magnetic field is stabilizing but weak so that the conditions are just to the left of $\mathscr{P}$, the mode with wave number $\alpha^2 = 4.218$ grows monotonically while other modes remain unstable. Now the constraining effects of salinity gradient and magnetic field are large enough to overcome the potential energy released by buoyancy force as well as viscous friction and at larger values all unstable modes to the left of $\mathscr{L}$ are direct. In other words, overstable modes are not possible when a sufficiently large combined salinity gradient and the magnetic field tends to cause the net upward diffusive density flux through oscillatory disturbances. Direct modes, however, can decrease the system potential energy by preferential transport of components with lower diffusivity. On the other hand direct instability cannot occur with $\partial\rho/\partial z < 0$ if R is sufficiently small compared with Q. The subsequent finite amplitude motions are considered in the next section.

6. FINITE AMPLITUDE ANALYSIS WITH LIMITED REPRESENTATION

In this section we discuss the finite amplitude analysis by considering a truncated representation [27] of velocity, magnetic, temperature and concentration fields. Such a study is very useful to understand the physics of the problem with minimum mathematics and the results can also be used as the starting values while discussing the general non-linear problem.

The first effect of non-linearity is to distort the temperature field through the interaction of ψ, ϕ, and T, concentration field through the interaction of ψ and S and the zonal current field through the interaction of ψ and ϕ. The distortion of temperature and concentration fields corresponds to a change in the horizontal mean, i.e. a component of the form $\sin 2\pi z$ will be generated. Similarly, the zonal current field will be distorted by a component of the form $\sin 2\pi\alpha x$. Thus a minimal system which describes finite amplitude convection is given by

$$\psi = A(t)\sin \pi\alpha x \sin \pi z \quad (25)$$

$$T = B(t)\cos \pi\alpha x \sin \pi z + C(t)\sin 2\pi z \quad (26)$$

$$S = D(t)\cos \pi\alpha x \sin \pi z + E(t)\sin 2\pi z \quad (27)$$

$$\phi = F(t)\sin \pi\alpha x \cos \pi z + G(t)\sin 2\pi\alpha x \quad (28)$$

where the amplitudes A, B, C, D, E, F and G are in general functions of time t and position which are to be determined by the dynamics of the system. Substituting equations (25)–(28) into equations (3)–(6) and equating the coefficients of like terms and assuming the amplitudes are steady (i.e. $\partial/\partial t = 0$) we get the following set of equations as the deterministic set for the steady amplitudes

$$k^4A + Q\tau_2\pi k^2F + R\pi\alpha B - R_s\pi\alpha D + Q\tau_2\pi^4\alpha(1-3\alpha^2)FG = 0 \quad (29)$$

$$\tau_2k^2F - \pi A - \pi^2\alpha AG = 0 \quad (30)$$

$$4\pi^2\alpha^2\tau_2G + \tfrac{1}{2}\pi^2\alpha AF = 0 \quad (31)$$

$$k^2B + \pi\alpha A + \pi^2\alpha AC = 0 \quad (32)$$

$$-4C + \tfrac{1}{2}\alpha AB = 0 \quad (33)$$

$$\tau_1k^2D + \pi\alpha A + \pi^2\alpha AE = 0 \quad (34)$$

$$-4\tau_1E + \tfrac{1}{2}\alpha AD = 0. \quad (35)$$

These steady solutions are very useful because they predict that a finite amplitude solution to the system is possible for subcritical values of the Rayleigh number R and that the minimum values of R for which a steady solution is possible lies below the critical values for instability to either a marginal state or an overstable infinitesimal perturbation. Elimination of all amplitudes, except A, yields (after some algebraic manipulation)

$$A\left\{\left(\frac{A^2}{8}\right)^4 + \left(\frac{k^2}{\pi^2\alpha^2}\right)l_0\left(\frac{A^2}{8}\right)^3 + \left(\frac{k^2}{\pi^2\alpha^2}\right)^2 l_1\left(\frac{A^2}{8}\right)^2 + \left(\frac{k^2}{\pi^2\alpha^2}\right)^3 l_2\left(\frac{A^2}{8}\right)^3 + \tau_1^2\sigma^{*2}\left(\frac{k^2}{\pi^2\alpha^2}\right)^4 l_3\right\} = 0 \quad (36)$$

where

$$l_0 = 2\sigma^*\left(1 + \frac{2\lambda m}{\tau_2}(1+\alpha^{-2})^{-1}\right) - \lambda + \lambda_s\tau_1 + 1 + \tau_1^2$$

$$l_1 = \sigma^*(\sigma^* + 2(1+\tau_1^2+\lambda_s\tau_1-\lambda)) + \tau_1^2(\lambda_s/\tau_1 - \lambda + 1) + 4\frac{\lambda_m\sigma^*}{\tau_2}((1+\tau_1^2)(1+\alpha^{-2})^{-1} + \sigma^*/4),$$

$$l_2 = \sigma^*(2\tau_1^2 + \sigma^* + 2\tau_1^2(\sigma^*/2 + \lambda_s/\tau_1 - \lambda) + 4\frac{\lambda_m}{\tau_2}\tau_1^2(1+\alpha^{-2})^{-1} + \sigma^*(\lambda_m/\tau_2(1+\tau_1^2) + \lambda_s\tau_1 - \lambda))$$

$$l_3 = 1 - \lambda + \lambda_s/\tau_1 + \lambda_m/\tau_2$$

with

$$\sigma^* = \tau_2^2\alpha^2.$$

The solution $A = 0$ corresponds to pure conduction, which we know to be a possible solution though it is unstable when R is sufficiently large. The remaining solutions are determined by solving the biquadratic equation numerically. Once we know the amplitude we can find the corresponding heat transport. It is interesting to note that when $\tau_2 = 0$, equation (36) reduces to the one given by Rudraiah *et al.* [9].

6.1. *Heat transport by convection*

In the study of double-diffusive magnetoconvection, the onset of convection as the Rayleigh number is increased is more readily detected by its effect on the heat transfer. In the quiescent state, the heat transfer is due to conduction (radiative heat transfer is usually neglected). Hence if H is the rate of heat transfer per unit area

$$H = -\kappa\left\langle\frac{\partial}{\partial z}T_{\text{total}}\right\rangle_{z=0} \quad (37)$$

where the angular brackets correspond to a horizontal average with the definition of T_{total} given by equation (1). Equation (37) can be written in the form

$$H = \kappa\frac{\Delta T}{d} - \kappa\frac{\Delta T}{d}\sum_{n=1}^{N} n\pi b_{\text{on}} \quad (38)$$

with the restriction $N = 2$. The second term on the RHS of equation (38) represents the heat which enters at the bottom by conduction and is carried to the top by both conduction and convection, and hence heat transfer increases above that given by conduction alone. This process can be explained physically by the relationship between the driving temperature difference ΔT and the heat transport. In dimensionless variables this is the Rayleigh–Nusselt number curve. The Nusselt number Nu, is the ratio of the heat transported across any layer to the heat which would be transported by conduction alone. Thus from equation (38), the Nusselt number is

$$Nu = \frac{Hd}{\kappa\Delta T} = 1 - \sum_{n=1}^{N} n\pi b_{\text{on}} = 1 - 2\pi C \quad (39)$$

with $N = 2$. Similarly, the solute Nusselt number Nu_s is defined by

$$Nu_s = 1 - \sum_{n=1}^{N} n\pi c_{\text{on}} = 1 - 2\pi E \quad (40)$$

where B, C, D, E are given by equations (29)–(35). For Rayleigh numbers below the critical value, the heat transport is purely by conduction for which $A = 0$ and B, C, D, E, F and G are all zero. In that case equations (39) and (40) show that Nu and Nu_s have to be unity.

Our purpose in this section is to determine the effect of magnetic field on the heat (i.e. Nu) and mass (i.e. Nu_s) transport by thermal and mass advection terms for which $A \neq 0$. In ordinary viscous flow [30] the heat transport represented by Nu is a function of R_s and τ_1 only; whereas in double-diffusive magnetoconvection Nu is not only a function of R_s and τ_1 but also depends

Table 1(a) and (b). Nu_s and Nu vs R, τ_1 and τ_2 (with Nu_s the upper value in each pair) for $Q = 10^2$ and $R_s = 10^4$

(a) $R^c = 3.391 \times 10^4$	$\tau_1 = 0.32$		(b) $R^c = 1.003 \times 10^6$	$\tau_1 = 0.01$	
R	$\tau_2 = 0.5$	$\tau_2 = 0.1$	R	$\tau_2 = 0.5$	$\tau_2 = 0.1$
7.48×10^3	—	2.887542 2.264361	2.02×10^3	—	2.999646 1.721763
7.6×10^3	—	2.911421 2.376881	4.60×10^3	2.999629 1.700598	2.999944 2.565194
9.16×10^3	2.900070 2.321353	2.955518 2.636478	5.6×10^3	2.999934 2.503547	2.999957 2.646062
9.6×10^3	2.935452 2.50865	2.960096 2.668318	9.6×10^3	2.999973 2.761333	2.999977 2.796862
2.5×10^4	2.989889 2.905452	2.99055 2.911381	2.5×10^4	2.999992 2.918498	2.999992 2.922901
3.39×10^4	2.993085 2.934453	2.993397 2.937329	1.0×10^6	3.00000 2.998087	3.0000 2.998090
6.78×10^4	2.996854 2.969693	2.996919 2.970316	2.0×10^6	3.00000 2.999044	3.0000 2.9999045
1.02×10^5	2.997963 2.980280	2.99799 2.980545	3.0×10^6	3.0000 2.999363	3.0000 2.999363

on Q and τ_2 and the heat transport is independent of σ. In other words the magnetic field will affect the heat transport.

To know the quantitative effect of the magnetic field on heat transport we have computed Nu and Nu_s for different values of Q, τ_2 and τ_1 for a fixed value of R_s and the results are shown in Table 1(a) and (b) and also in Fig. 4. In Table 1(a) and (b) the values of Nu_s and Nu (with Nu_s the upper value in each pair) for $\tau_1 = 0.32$, 0.01, $R_s = 10^4$, $Q = 10^2$ and $\tau_2 = 0.5, 0.1$ are tabulated and they are independent of σ. In all these cases, $Nu > 1$ even for $R < R^c$. This establishes that subcritical motions are possible. These tables show that the amount of heat and solute convected by steady modes increase with R. Further, the decrease in Q increases the values of Nu and Nu_s (Fig. 4(a)). For a fixed value of Q, Nu and Nu_s increase as τ_2 decreases. In other words, a fluid having less magnetic diffusivity transports more heat and salt for a fixed value of Q (see Table 1(a) and (b)).

The results obtained for $Q = 10^4$, $\tau_2 = 0.5, 0.25$ with $\tau_1 = 0.32, 0.81$ and 0.01 for a fixed value of $R_s = 10^4$, are

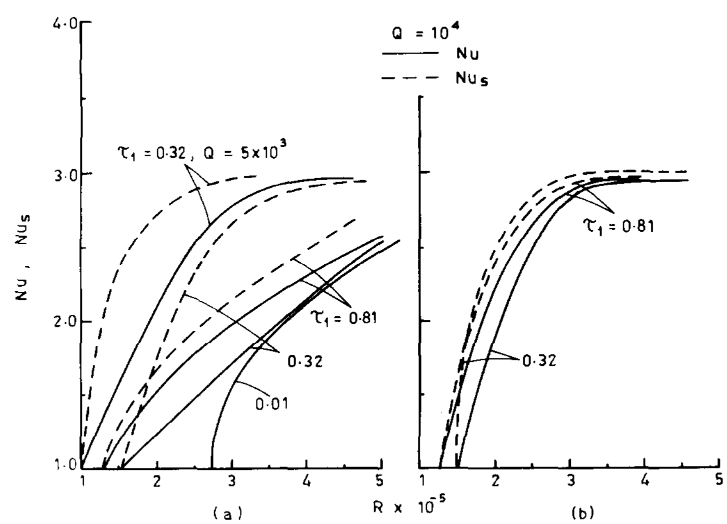

FIG. 4. Nu and Nu_s vs R for $R_s = 10^4$ at two values of τ_2: (a) $\tau_2 = 0.5$; (b) $\tau_2 = 0.25$.

depicted in Figs. 4(a) and (b). For a certain range of values of τ_1 and τ_2 subcritical motions are not possible. For example for the set $Q = 10^4, \tau_2 = 0.5, 0.25$ when $\tau_1 = 0.32$ and 0.81 for a fixed value of $R_s = 10^4$ subcritical motions are not possible. However, for other values of τ_2 and τ_1 the subcritical motions are possible. For example, for $\tau_2 = 0.1$ with $\tau_1 = 0.32, 0.81, 0.01$ and $\tau_2 = 0.5$, 0.25 with $\tau_1 = 0.01$ subcritical motions are possible. In all the cases we note that $Nu_s > Nu$. The heat and mass transport are computed for a comparatively weak magnetic field for example ($Q = 10^{-4}$, 10^{-2}) and we found that Nu and Nu_s are independent of τ_1 and τ_2. Even in the case of a weak magnetic field we found that subcritical motions are possible because of a strong concentration gradient.

7. RESULTS

The stability of a double-diffusive magneto-convection is investigated subjected to infinitesimal and finite amplitude disturbances. The infinitesimal disturbance analysis reveals that:

(a) The marginal stability of oscillatory modes occurs on a hyperboloid in $(R, R_s, Q\tau_2)$ space but the space is very closely approximated by its planar asymptotes for any diffusivity ratios. The effect of magnetic field is to make the system more stable. The region of salt-finger and overstable modes decrease as τ_1 increases.

(b) The combined effect of heat and magnetic field with the greatest and smallest (τ_2) diffusivity ratios, salt-finger and overstable modes may be simultaneously unstable over a wide range of conditions and the effect of magnetic field is to suppress this region.

From the study of finite amplitude analysis we can conclude that the finite amplitude steady convection should exist for subcritical values of R for all Q, τ_1 when $\tau_2 = 0.1$ for a fixed value of $R_s = 10^4$. The heat transport increases with an increase in R and a decrease in τ_2 and decreases with Q because the magnetic field inhibits the onset of convection.

Finally, we conclude that although the truncated representation adopted in this paper throws some physical insight on the stability boundaries and the effect of magnetic field on heat and mass transport it is silent about the influence of Prandtl number directly on the amplitude of double-diffusive magnetoconvection and hence on the heat and mass transport. To find the effect of Prandtl number explicitly we have to consider higher-order approximations. Work in this direction is in progress.

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CONVECTION DOUBLEMENT DIFFUSANTE AVEC UN CHAMP MAGNETIQUE IMPOSE

Résumé—On étudie analytiquement la magnétoconvection bidimensionnelle, linéaire à amplitude finie (convection thermohaline en présence d'un champ magnétique) avec des frontières libres horizontales maintenues à température et concentration fixes. On montre que le champ magnétique agit comme une troisième composante de diffusion et son effet est de supprimer la convection. Si $\tau_{\sim}$, rapport de la diffusivité magnétique à la diffusivité thermique, est petit et si le nombre de Rayleigh du soluté R_S et le nombre de Chandrasekhar Q sont suffisamment grands, la convection apparait avec des oscillations stables et l'apparition de ceci est approché par deux lignes droites dans le plan du nombre de Rayleigh. Dans le cas de la théorie non linéaire, on trouve que la magnétoconvection à amplitude finie existe pour des valeurs sous-critiques du nombres de Rayleigh R, pour tous Q et τ_1 (qui est le rapport de la diffusivité de soluté à la diffusivité thermique) quand $\tau_2 = 0{,}1$ et $R_S = 10^4$. On trouve que le transfert de chaleur croît quand R augmente et τ_2 décroît, mais diminue avec Q.

GEGENDIFFUSIONSKONVEKTION MIT ÜBERLAGERTEM MAGNETFELD

Zusammenfassung—Zweidimensionale Gegendiffusions-Magneto-Konvektion (thermohaline Konvektion in Anwesenheit eines Magnetfeldes) wurde analytisch mit Hilfe der linearen Theorie und der Theorie endlicher Amplitude untersucht, wobei freie horizontale Berandungen konstanter Temperatur und Konzentration vorausgesetzt wurden. Es wird gezeigt, daß das Magnetfeld als dritte treibende Diffusionskraft wirkt und dabei die Konvektion behindert. Im Falle der linearen Theorie wurden die Bedingungen für stationäre und oszcillierende Betriebsweise betrachtet und die Stabilitätsgrenzen für Salzfinger und Gegendiffusions-Konvektion in der Rayleigh-Zahlenebene vorausberechnet. Ist τ_2, das Verhältnis von magnetischer und Temperatur-Leitfähigkeit, klein und die Rayleigh-Zahl des gelösten Stoffes R_s sowie die Chandrasekhar-Zahl Q ausreichend groß, so setzt Konvektion als überstabile Schwingung ein. Die Grenzen hierfür lassen sich in der Rayleigh-Zahlenebene durch zwei Geraden annähern. Es wurde herausgefunden, daß der Salzfinger und der überstabile Zustand gleichzeitig über einen weiten Bereich der Bedingungen instabil sein können, wobei der Einfluß des Magnetfeldes diesen Bereich einschränkt. Im Falle der nichtlinearen Theorie ergibt sich Magneto-Konvektion mit einer endlichen Amplitude für unterkritische Werte der Rayleigh-Zahl R sowie für alle Q und alle τ_1 (das Verhältnis von Lösungs- und Temperatur-Leitfähigkeit), wenn $\tau_2 = 0{,}1$ und $R_s = 10^4$ ist. Es zeigte sich, daß sich der Wärmetransport erhöht, wenn R ansteigt und τ_2 fällt, aber abnimmt, wenn Q fällt.

ДВОЙНАЯ ДИФФУЗИОННАЯ КОНВЕКЦИЯ ПРИ ВОЗДЕЙСТВИИ МАГНИТНОГО ПОЛЯ

Аннотация—Аналитически исследована в линейном приближении конечной амплитуды двумерная двойная диффузионная магнитная конвекция (термохалинная конвекция в присутствии магнитного поля) в случае, когда свободные горизонтальные границы поддерживаются при постоянной температуре и постоянной концентрации. Показано, что магнитное поле действует как третий диффундирующий компонент, который подавляет конвекцию. В случае линейной теории определены условия для устойчивого и колебательного режимов течения и рассчитаны границы устойчивости конвекции для солевой двойной диффузионной конвекции в плоскости чисел Рэлея. При малом значении τ_2, представляющем отношение коэффициента магнитной диффузии к коэффициенту температуропроводности, и достаточно больших значениях чисел Рэлея R_S и Чандрасекхара Q, рассчитанных для растворенного вещества, возникают сверхустойчивые колебания, начало которых аппроксимируется двумя прямыми линиями в плоскости чисел Рэлея. Найдено, что режим солевой конвекции и сверхустойчивый режим могут быть одновременно неустойчивыми в широком диапазоне условий, и магнитное поле уменьшает эту зону. На основе нелинейной теории установлено, что магнитная конвекция конечной амплитуды имеет место при докритических значениях числа Рэлея R для всех значений Q и τ_1 (отношение коэффициента диффузии растворенного вещества к коэффициенту температуропроводности), когда $\tau_2 = 0.1$ и $R_S = 10^4$. Найдено, что величина теплового потока растет с увеличением R и уменьшением τ_2, но уменьшается с уменьшением Q.